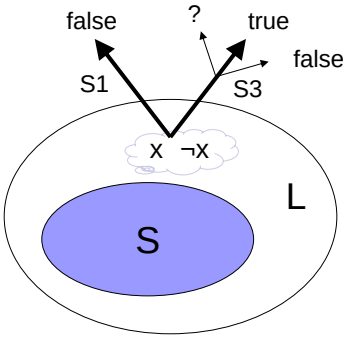
Cadoli-Schaerf Approximation

Anytime Algorithms for logical entailment

- State of the Art: S1-/S3-entailment
  - sound and complete
  - semantic approach
- S1-entailment: interpret everything outside of S as false
- S3-entailment: interpret everything outside of S as true



S3-Approximation for Description Logics (ALE)

- Function for computing the approximated conceptterm according to S_i

$$(\exists \text{friend.tall}) \sqcap \forall \text{friend.}((\forall \text{friend.doctor}) \sqcap \exists \text{friend.}\neg\text{doctor}).$$

  - Levels i = nested quantifiers

L_1 $(\forall \text{friend.doctor}) \sqcap \exists \text{friend.}\neg\text{doctor}$

L_0 $(\exists \text{friend.tall}) \sqcap \forall \text{friend.}(\quad)$.

  - $S_i^\top$ = Ommit all exist qantifiers greater or equal level i

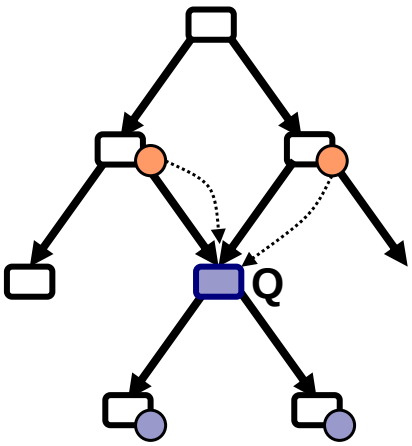
$S_0^\top$ $\top \sqcap \forall \text{friend.}((\forall \text{friend.doctor}) \sqcap \top)$

$S_1^\top$ $(\exists \text{friend.tall}) \sqcap \forall \text{friend.}((\forall \text{friend.doctor}) \sqcap \top)$

$S_2^\top$ $(\exists \text{friend.tall}) \sqcap \forall \text{friend.}((\forall \text{friend.doctor}) \sqcap \exists \text{friend.}\neg\text{doctor}).$

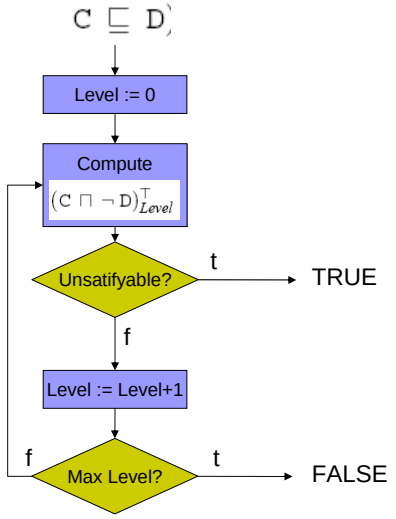
$$\exists R.C \mapsto \top$$

Application: Individual Retrieval



- Retrieval Process
  - Classify Query Q
  - Select Instances from subsumed classes
  - Realize instances from direct parents, if the belongs to Q
- Cmp. Instance Store for role-free A-Boxes

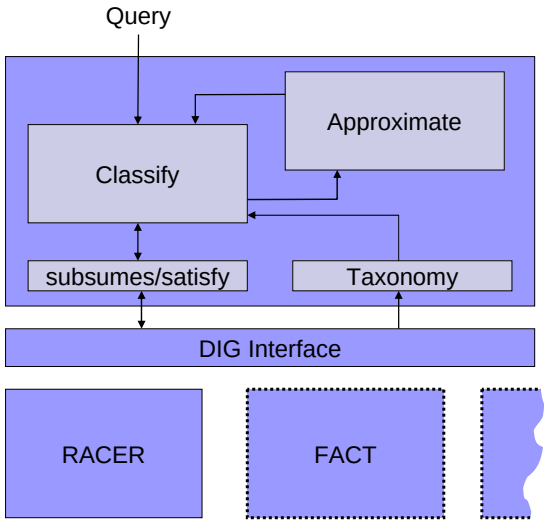
Approximating the Classification



- *C is subsumed by D* ⇔
  $C \sqcap \neg D$ *is unsatisfiable*
- Cadoli-Schaerf ensures:

$(C \sqcap \neg D)^{\top}_{Level}$ is unsatisfiable
$\Rightarrow (C \sqcap \neg D)$ is unsatisfiable

Implementation



Done

- Implemented test bed for trials
  - ALE-Approximation implemented
  - Approximation analysis implemented
  - In Prolog
  - Uses and extends VU's DIG-Interface
  - Racer used as DLR
  - Logging (Time, Memory)
  - Spreadsheet for analyzing Logging

Results …

… Sorry no results at the moment
but many problems

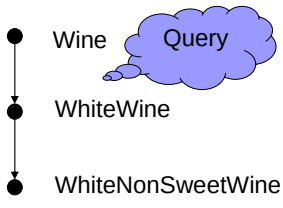
Problem I: Several definitions

- Concepts must not defined by one axiom
- A concept can be defined by several axioms
- Mixture of equivalence and inclusion axioms allowed

- Which concept definition should be used for the approximation?

Example: Wine Ontology (I)

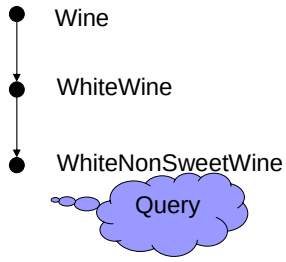
$$\begin{aligned} \texttt{WhiteWine} &\equiv \texttt{Wine} \sqcap \exists \text{hasColor}.\{\mathit{White}\}. \\ \texttt{WhiteNonSweetWine} &\equiv \texttt{WhiteWine} \sqcap \forall \text{hasSugar}.\{\mathit{OffDry}, \mathit{Dry}\}, \\ \texttt{CheninBlanc} &\equiv \texttt{Wine} \sqcap \\ &\quad \leq 1\, \text{madeFromGrape}.\top \sqcap \\ &\quad \exists \text{madeFromGrape}.\{\mathit{CheninBlancGrape}\}, \end{aligned}$$



$\texttt{Query} \sqsubseteq \texttt{CheninBlanc}$

Example: Wine Ontology (II)

$$\begin{aligned}
\texttt{WhiteWine} &\equiv \texttt{Wine} \sqcap \exists \texttt{hasColor}.\{\mathit{White}\}.\\
\texttt{WhiteNonSweetWine} &\equiv \texttt{WhiteWine} \sqcap \forall \texttt{hasSugar}.\{\mathit{OffDry}, \mathit{Dry}\},\\
\texttt{CheninBlanc} &\equiv \texttt{Wine} \sqcap\\
&\quad \leq 1\,\texttt{madeFromGrape}.\top \sqcap\\
&\quad \exists \texttt{madeFromGrape}.\{\mathit{CheninBlancGrape}\},\\
\texttt{CheninBlanc} &\sqsubseteq \forall \texttt{hasSugar}.\{\mathit{OffDry}, \mathit{Dry}\} \sqcap\\
&\quad \exists \texttt{hasColor}.\{\mathit{White}\} \sqcap\\
&\quad \exists \texttt{hasFlavor}.\{\mathit{Moderate}\} \sqcap\\
&\quad \forall \texttt{hasBody}.\{\mathit{Medium}, \mathit{Full}\}.
\end{aligned}$$



$\texttt{Query} \sqsubseteq \texttt{CheninBlanc}$

Idea: Combine several definitions (I)

1. Convert inclusion axioms into equivalence axioms

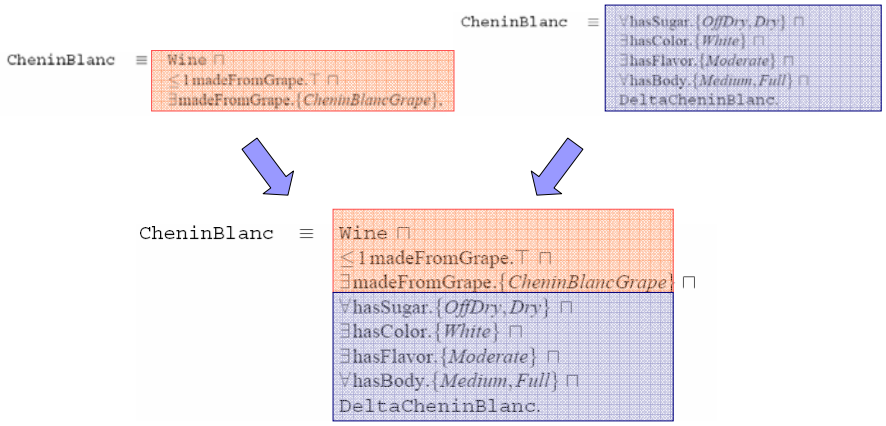
$$\begin{aligned}
\texttt{CheninBlanc} &\sqsubseteq \forall \texttt{hasSugar}.\{\mathit{OffDry}, \mathit{Dry}\} \sqcap\\
&\quad \exists \texttt{hasColor}.\{\mathit{White}\} \sqcap\\
&\quad \exists \texttt{hasFlavor}.\{\mathit{Moderate}\} \sqcap\\
&\quad \forall \texttt{hasBody}.\{\mathit{Medium}, \mathit{Full}\}.
\end{aligned}$$

$$\begin{aligned}
\texttt{CheninBlanc} &\equiv \forall \texttt{hasSugar}.\{\mathit{OffDry}, \mathit{Dry}\} \sqcap\\
&\quad \exists \texttt{hasColor}.\{\mathit{White}\} \sqcap\\
&\quad \exists \texttt{hasFlavor}.\{\mathit{Moderate}\} \sqcap\\
&\quad \forall \texttt{hasBody}.\{\mathit{Medium}, \mathit{Full}\} \sqcap\\
&\quad \texttt{DeltaCheninBlanc}.
\end{aligned}$$

Additional primitive concept representing the delta

Idea: Combine several definitions (II)

2. Conjunct all equivalence axioms



But …

- Information lost:

$$\texttt{Wine} \sqcap \leq 1\,\texttt{madeFromGrape}.\top \sqcap \exists\,\texttt{madeFromGrape}.\texttt{Grape} \equiv \forall\,\texttt{hasSugar}.\{\mathit{OffDry}, \mathit{Dry}\} \sqcap \exists\,\texttt{hasColor}.\{\mathit{White}\} \sqcap \exists\,\texttt{hasFlavor}.\{\mathit{Moderate}\} \sqcap \forall\,\texttt{hasBody}.\{\mathit{Medium}, \mathit{Full}\}\ \texttt{DeltaCheninBlanc}.$$

- Solution:
  - Conjunct only virtually (i.e., during the approximate classification)
  - Add a general axiom

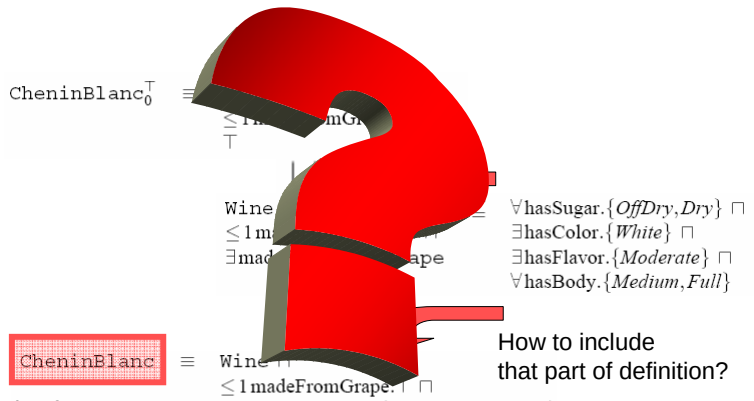
Problem II: General Axioms

- Combining several axioms only works if they can be grouped

$$\begin{aligned}
\texttt{WhiteWine} &\equiv \texttt{Wine} \sqcap \exists \texttt{hasColor}.\{\mathit{White}\}.\\
\texttt{WhiteNonSweetWine} &\equiv \texttt{WhiteWine} \sqcap \forall \texttt{hasSugar}.\{\mathit{OffDry}, \mathit{Dry}\},\\
\texttt{CheninBlanc} &\equiv \texttt{Wine} \sqcap \\
&\quad \leq 1\,\texttt{madeFromGrape}.\top \sqcap \\
&\quad \exists \texttt{madeFromGrape}.\{\mathit{CheninBlancGrape}\},
\end{aligned}$$

$$\begin{aligned}
\texttt{Wine} \sqcap & \\
\leq 1\,\texttt{madeFromGrape}.\top \sqcap & \\
\exists \texttt{madeFromGrape}.\texttt{Grape} &
\end{aligned} \quad \sqsubseteq \quad
\begin{aligned}
&\forall \texttt{hasSugar}.\{\mathit{OffDry}, \mathit{Dry}\} \sqcap \\
&\exists \texttt{hasColor}.\{\mathit{White}\} \sqcap \\
&\exists \texttt{hasFlavor}.\{\mathit{Moderate}\} \sqcap \\
&\forall \texttt{hasBody}.\{\mathit{Medium}, \mathit{Full}\}
\end{aligned}$$

Open Problems



Also for Query possible?

Problem III: Practical useless approximation

$\Leftrightarrow$ (Query $\sqsubseteq$ WhiteNonSweetWine) holds.
$\Leftrightarrow$ (Query $\sqcap$ $\neg$WhiteNonSweetWine) is unsatisfiable
$\Leftarrow$ (Query $\sqcap$ $\neg$WhiteNonSweetWine)$_0^\top$ is unsatisfiable
$\Leftrightarrow$ Query$_0^\top$ $\sqcap$ ($\neg$WhiteNonSweetWine)$_0^\top$ is unsatisfiable

$\exists R.C \mapsto \top$

$$\begin{aligned}
\texttt{WhiteNonSweetWine} &\equiv \texttt{Wine} \sqcap \exists \text{hasColor}.\{\mathit{White}\} \sqcap \forall \text{hasSugar}.\{\mathit{OffDry}, \mathit{Dry}\} \\
\neg\texttt{WhiteNonSweetWine} &\equiv \neg\texttt{Wine} \sqcup \forall \text{hasColor}.\neg\{\mathit{White}\} \sqcup \exists \text{hasSugar}.\neg\{\mathit{OffDry}, \mathit{Dry}\}. \\
(\neg\texttt{WhiteNonSweetWine})_0^\top &\equiv \neg\texttt{Wine} \sqcup \forall \text{hasColor}.\neg\{\mathit{White}\} \sqcup \top \\
&\equiv \top.
\end{aligned}$$

! Many concept terms are approximated to $\top$

$\Leftrightarrow$ Query$_0^\top$ $\sqcap$ $\top$ is unsatisfiable
Query$_0^\top$ is unsatisfiable

Example: Wine Ontology

- 1 element: #58
- 2 elements: #1
- 9 elements: #1 (top)

Very bad partitioning!

Procedure:

- Estimate computational power
  - For every concept definition
    - Approximate and
    - Classify it
  - Extract the equivalents (Clusters)
- Definition should be partitioned into few clusters of nearly the same size

Problem Summary

- Problem I & II:
  - Approximation seems to be only possible where
    - general axioms are forbidden and
    - Only one definition for a concept

    } "Old style" description logics
  - Independent from the approximation function
- Problem III:
  - Dependent from the approximation function
  - Find better approximation function

TODO

- Solve the problems
- Different Approximation methods (ALC, own developed!)
- Different ontologies (large one)
- Fact vs Racer ;-)
- Different Approximation strategy (classification by "large steps")
- Estimating Time for approximation